

Time Allowed: 3 hrs
Date: 8/1/2017

Note: Assume any data required, state your assumption clearly. Answer all the following Questions

## Question (1)

(30 Marks)
A property $\phi$ is transported by means of convection and diffusion through the one dimensional domain sketched in Fig. 1. The governing equation is $\frac{d \rho u \phi}{d x}=\frac{d}{d x}\left(\Gamma\left(\frac{d \phi}{d x}\right)\right)$ the boundary conditions are $\phi_{o}=1.0$ at $\mathrm{x}=0$ and $\phi_{L}=0.0$ at $\mathrm{x}=\mathrm{L}$. Using five equally spaced cells and the upwind differencing scheme, calculate the distribution of $\phi$ as a function of $x$. The following data apply $u=0.1 \mathrm{~m} / \mathrm{s}$, length $\mathrm{L}=1.0 \mathrm{~m}, \rho=1.0 \mathrm{~kg} / \mathrm{m}^{3}, \Gamma=0.1 \mathrm{~kg} / \mathrm{m} . \mathrm{s}$.

## Question (2)

(30 Marks)
In Fig. 2 a two- dimensional plate of thickness 1 cm is shown. The governing equation is $\frac{\partial}{\partial x}\left(k\left(\frac{\partial T}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(k\left(\frac{\partial T}{\partial y}\right)\right)=0.0$. The thermal conductively of a plate material is $\mathrm{k}=1000$ $\mathrm{W} / \mathrm{m} . \mathrm{K}$. The west boundary receives a steady heat flux of $500 \mathrm{~kW} / \mathrm{m}^{2}$ and the south and east boundaries are insulated. If the north boundary is maintained at a temperature of $100^{\circ} \mathrm{C}$, use a uniform grid with $\Delta x=\Delta y=0.1 \mathrm{~m}$ to calculate the steady state temperature distribution at nodes


Fig. 1
Fig. 2

## Question (3)

(40 Marks)
The $\boldsymbol{x}$ - component of Navier-Stokes equation for two-dimensional porous media with no body force can be written as: $\frac{\partial \rho u^{2}}{\partial x}+\frac{\partial \rho u v}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)-k\left(u^{2}+v^{2}\right)$
Where last term represent additional resistance due to porous media
a) Drive the finite volume difference equation over a staggered grid and show how the under-relaxation affects the coefficient of the obtained equation.
b) Drive an expression for pressure correction equation using SIMPLE algorithm.
c) How the source term be linearized?
d) Draw a flow chart for the solution of 2-D fluid flow in straight duct.

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